VP160 Recitation Class

FANG Yigao

June 8, 2020

Harmonic Oscillator with Linear Damping

Driven Oscillation

Non-inertial frames of reference

Earth as a Frame of Reference

Work

Harmonic Oscillator with Linear Damping

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

1. Overdamped: $b^2 > 4km$
   $x(t) = C_1 e^{-(\frac{b}{2m}+\sqrt{\frac{b^2}{4m^2}-\omega_0^2})t} + C_2 e^{-(\frac{b}{2m}-\sqrt{\frac{b^2}{4m^2}-\omega_0^2})t}$
2. Critically damped: $b^2 = 4km$
   $x(t) = C_1 e^{-\frac{b}{2m}t} + C_2 t e^{-\frac{b}{2m}t}$
3. Underdamped: $b^2 < 4km$
   $x(t) = e^{-\frac{b}{2m}t}[Acos(\sqrt{\omega_0^2 - \frac{b^2}{4m^2}}t) + Bsin(\sqrt{\omega_0^2 - \frac{b^2}{4m^2}}t)]$

Driven Oscillation

Equations

$$\ddot{x}(t) + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m} cos\omega_{dr} t$$

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_{dr}^2)^2 + (\frac{b\omega_{dr}}{m})^2}}$$

$$tan\phi = \frac{b\omega_{dr}}{m(\omega_{dr}^2 - \omega_0^2)}$$

$$\omega_{res} = \sqrt{\omega_0^2 - \frac{b^2}{2m^2}}$$

Reminders

1. The resonance frequency is lower than the natural frequency if we have drag.
2. The response of the system is not in phase with the drive.

Non-inertial frames of reference

Rotating Frame of Reference

$$m\vec{a'} = \vec{F} - m\vec{a_{O'}} - m\frac{d\vec{\omega}}{dt} \times \vec{r'} - 2m(\vec{\omega} \times \vec{v'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

Derivation Process

How to derive this formula?

Non-inertial frames of reference

Pseudo Forces

Acceleration	Name	Resource
$\vec{a_{O'}}$	d'Alembert	acceleration of O'
$\frac{d\vec{\omega}}{dt} \times \vec{r'}$	Euler	angular acceleration of O'
$2\vec{\omega} \times \vec{v'}$	Coriolis	rotation of O' and motion in O'
$\vec{\omega} \times (\omega \times \vec{r'})$	centrifugal	rotation of O'

Basic Equation

$$m\vec{a'} = \vec{F} - m\vec{a_{O'}} - m\frac{d\vec{\omega}}{dt} \times \vec{r'} - 2m(\vec{\omega} \times \vec{v'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

Comments

Most of the time, earth is considered an inertial frame of reference, since the rotation of earth is quiet slow and can be ignored. However, there are some exceptions when it comes to Coriolis force, for example, Foucault pendulum.

Work

Defintion

$$\delta W = \vec{F} \circ d\vec{r} \tag{1}$$

$$W_{AB} = \int_{AB} \vec{F} \circ d\vec{r} \tag{2}$$

Methods for Calculation

1. Constant force on a straight line
2. Varying force on a straight line (Integral)
3. Varying force on a curve (Line integral)

Kinetic Energy and Power

Kinetic Energy

Kinetic energy: $E_k = \frac{1}{2}mv^2$

Work-Kinetic Energy Theorem: $\delta W = dE_k$

Power

Average power: $P_{av} = \frac{W}{\Delta t}$

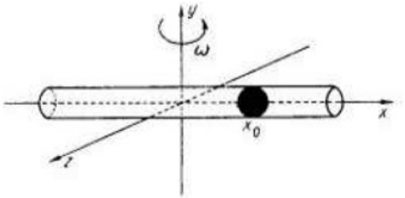
Instantaneous power: $P_{is} = \frac{dW}{dt} = \vec{F} \circ \vec{v}$

Exercise 1

A box is filled with a liquid and is placed on a horizontal surface. Find the angle that the surface of the liquid forms with the horizontal if we pull the box with acceleration a.

Exercise 2

A particle with mass m is inside a pipe that rotates with constant angular velocity ω about an axis perpendicular to the pipe.The kinetic coefficient of friction is equal to μ_k. Write down (do not solve!) the equation of motion for this particle in the non-inertial frame of reference of the rotating pipe. There is no gravitational force in this problem.



Exercise 3

A uniform cylinder of mass m, radius R, and height h is roating vertically in a liquid, so that it is half-immersed in the liquid. Find the density of the liquid and minimum work needed to pull the cylinder completely above the liquids surface.

Exercise 4

If we let go an object 100m above the equator. Ignore the air drag, calculate the deviation caused by Coriolis Force.